On the rank of appearance of polynomial Lucas sequences

Joaquim Cera Da Conceição

Laboratoire de Mathématiques Nicolas Oresme, Caen

We study the distribution of primes whose rank of appearance in Lucas sequences is divisible by a fixed integer. The problem over the integers was first considered by Hasse in 1965, with examples such as the sequence $2^n - 1$, which has Dirichlet density $17/24$ of primes with even rank. A complete characterisation is known when the sequence has a reducible characteristic polynomial, due to Wiertelak. In a recent work, Sanna obtained density results in the irreducible case. We adapt these methods to polynomial Lucas sequences over a finite field. In this setting, we prove density results with respect to a stronger notion of density than the Dirichlet density. Our results hold in full generality, except for the computation of explicit formulas for the density.