

## Rational angles in plane lattices

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A lattice is a discrete subgroup of  $\mathbb{R}^2$  isomorphic to  $\mathbb{Z}^2$ . It is a natural question to investigate all ‘nice’ configurations of points that can be found in a lattice, such as regular polygons with all their vertices on lattice points. In joint work with Roberto Dvornicich, Francesco Veneziano and Umberto Zannier, we classify more generally all plane lattices whose points form three or more (genuinely distinct) angles with amplitude a rational multiple of  $\pi$ . The underlying number-theoretic problem is a Diophantine equation in several unknowns, some of which are constrained to be rational numbers, while others are roots of unity. To solve this equation, we combine the general theory of vanishing sums of roots of unity, a Galois-theoretic technique to reduce the resulting bounds, and a direct study of rational points on several curves of genus up to 5.