Specializing quaternionic big Heegner points in Hida families

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Central objects in algebraic number theory are Galois representations. To a suitable Galois representation one can attach two fundamental objects: an *L-function*, a special complex function, and a *Selmer group*, an algebraic object that contains deep number theoretical information. Even though these two objects have really different natures, they are related by the *Bloch-Kato Conjecture* that predicts, at least in the cases of our interest, an equality between the order of vanishing of the L-function at a certain point and the dimension of the Selmer group. We are interested in Galois representations associated with modular forms and Hida families of modular forms.

One of the most powerful instruments for studying Selmer groups of Galois representations associated with modular forms, in particular in order to obtain results towards the Bloch-Kato Conjecture, are Euler systems of *Heegner cycles* or *Heegner points*, that are special collections of certain geometric objects. The p-adic variation of Heegner points on Hida families has been considered by Howard [4], who defined the notion of big Heegner points attached to the big Galois representation associated with a Hida family of modular forms and a quadratic imaginary field K satisfying the so-called *Heegner hypothesis* with respect to the tame level of the family. The relation between Howard big Heegner points and (generalized) Heegner cycles associated with each form in the family ([1]), has been discussed in several works, in particular by Castella ([3]), who proves an explicit relation between them.

In this talk, we see that this relation can be generalized in the so-called quaternionic setting, where the imaginary quadratic field satisfies a weaker hypothesis, called *generalized Heegner hypothesis*. In this case, the construction of big Heegner points has been done by Longo-Vigni [5], while generalized Heegner cycles has been constructed by Brooks [2], in both cases replacing modular curves with Shimura curves attached to indefinite rational quaternion algebras. As in [3], the key point is to use the theory of p-adic L-functions as a bridge between these two families, building a big p-adic L-function which interpolates the p-adic L-functions associated with the modular forms of the family. This is a joint work with Matteo Longo and Eduardo Rocha Walchek.

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[2] E. H. Brooks: "Shimura curves and special values of p-adic L-functions", Int. Math. Res. Not. IMRN **12**, pp. 4177–4241, 2015.

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[4] B. Howard, "Variation of Heegner points in Hida families", Invent. Math. **167**, no. 1, pp. 91–128, 2007.

[5] M. Longo and S. Vigni, "Quaternion algebras, Heegner points and the arithmetic of Hida families", Manuscripta Math. **135**, 2011.