Abelian points in backward orbits of Lattés maps

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It is a standard result in Complex Multiplication Theory that given an elliptic curve E with complex multiplication by a quadratic number field F and $j(E) \neq 0,1728$, the x-coordinates of the torsion points of E generate an abelian extension of F(j(E)). This statement can be reformulated via Lattés maps; it says precisely that under certain conditions, given a number field K, a point $P \in K$ and a Lattés map $\phi \in K(x)$, the backward orbit of P generate an abelian extension of K. In this talk, we show that these conditions are also necessary. In other words, we characterize the triples (ϕ, P, K) with K a number field, $\phi \in K(x)$ a Lattés map and $P \in K$ such that the backward orbit of P via ϕ is abelian over K. Joint work with Ostafe and Zannier.