Continued fractions with extraneous denominators

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Given a number field K and a prime ideal $\mathfrak{P}$ of K lying over an odd prime p, each element $\alpha \in K$ can be expressed as a (possibly infinite) continued fraction

$$\alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{\ddots}}$$

where $a_0, a_1, \dots$ are $\mathfrak{Q}$-adic integers for each prime ideal $\mathfrak{Q}$ not lying over p. These CF-expansions can be computed by means of suitable algorithms, called *CF-algorithms*. It is natural to ask for which choices of K there exists a CF-algorithm such that the CF-expansion of each element in K is finite (*CFF property*). Previous works show that a major obstruction to the CFF property comes from the very structure of the class group of K. In this talk we circumvent this obstruction by giving a more general definition of CF-expansion: namely, we relax the requirement on $a_0, a_1, \dots$ by allowing a finite set of "extraneous" integers (i.e. other than p) to appear in their denominators. Our main result is that, for any K, a suitable choice of extraneous denominators ensures the CFF property for each prime p larger than some (effective) lower bound. Finally, we point out possible approaches to explicitly construct the corresponding CF-algorithms.

Joint work with L. Capuano, S. Checcoli and L. Terracini.