An average version of the Cilleruelo's conjecture for families of S_n -polynomials

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For $f \in \mathbb{Z}[X]$ an irreducible polynomial of degree n, the Cilleruelo's conjecture states

$$\log(\operatorname{lcm}(f(1),\ldots,f(M))) \sim (n-1)M\log M$$

as $M \to +\infty$, where lcm $(f(1), \ldots, f(M))$ is the least common multiple of $f(1), \ldots, f(M)$. It's well-know for n = 1 as a consequence of the Dirichlet's theorem for primes in arithmetic progression, and it was proved by Cilleruelo for degree-2 polynomials. Recently the conjecture was shown for a large family of polynomials of any degree by Rudnick and Zehavi. We want to investigate the case of S_n -polynomials with coefficients in the ring of algebraic integers of a fixed number field extension K/\mathbb{Q} in by considering the least common multiple of ideals of \mathcal{O}_K .