The multiplicative inverses modulo a Fibonacci number in terms of the Zeckendorf representation

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Let $(F_n)_{n\geq 1}$ be the Fibonacci sequence defined by the recurrence $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$, with $F_1 = F_2 = 1$. Every positive integer n can be written as a sum of distinct non-consecutive Fibonacci numbers, that is, $n = \sum_{i=1}^{m} d_i F_i$, where $m \in \mathbb{N}$, $d_i \in \{0, 1\}$, and $d_i d_{i+1} = 0$ for all $i \in \{1, \ldots, m-1\}$.

This is called the *Zeckendorf representation* of n and, apart from the equivalent use of F_1 instead of F_2 or vice versa, is unique.

In the literature, it is possible to find the Zeckendorf representation of numbers of the form F_{kn}/F_n, F_n^2/d, L_n^2/d and mF_n, where L_n are the Lucas numbers and d is a Lucas or Fibonacci number. Prempreesuk et al. determined the Zeckendorf representation of the multiplicative inverse of 2 modulo F_n, for every positive integer n not divisible by 3, where F_n denotes the nth Fibonacci number.

In this talk, we determine the Zeckendorf representation of the multiplicative inverse of a modulo F_n, for every fixed integer $a \geq 3$ and for all positive integers n with $\gcd(a, F_n) = 1$. Our proof makes use of the so-called base-φ expansion of real numbers.