On the polynomial Pell equation

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The classical Pell equation $A^2 - DB^2 = 1$, to be solved in integers A, B with $B \neq 0$ has a natural analogue in polynomials which goes back to Abel. In this setting, it is not always true that, if a polynomial $D \in \mathbb{C}[x]$ is not a square, then the corresponding Pell equation is solvable, and it seems that the set of polynomials for which there exists a nontrivial solution to the associated Pell equation (which we call Pellian) is rather "sparse". In the case of one parameter families of polynomials, D. Masser and U. Zannier gave a complete characterization of the families for which there exist infinitely many Pellian specializations. In a join work with F. Barroero and U. Zannier, we consider the "moduli space" of all monic polynomials of fixed even degree $2d \geq 4$ and prove, among other things, that the locus of Pellian polynomials consists of a denumerable union of subvarieties of dimension at most $d + 1$.