Irreducibility of random polynomials of large degree

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Let $f(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1} + x^n$ be a random monic polynomial, where a_j is chosen uniformly at random from $\{0,1\}$ and independently of the other coefficients. In 1993, Odlyzko and Poonen conjectured that f(x) is irreducible with probability $\sim 1/2$ when $n \to \infty$. Breuillard and Varjú proved that this expectation is indeed true under the Generalized Riemann Hypothesis. In this talk, I will present joint work with Bary-Soroker and Kozma that proves that f(x) is irreducible with probability $\geq 1/1000$ for all large enough n. In addition, if we condition on the event that f(x) is irreducible, then we prove that the Galois group of f(x) contains the alternating group A_n with conditional probability ~ 1 . The proofs use a fun mixture of ideas from sieve methods, the arithmetic of polynomials over finite fields, p-adic Fourier analysis, primes with restricted digits, Galois theory and group theory.