

**On the hyperbolic prime number theorem**

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Friedlander and Iwaniec established upper and lower bounds for the number of primes  $p = a^2 + b^2 + c^2 + d^2$  up to  $x$  with the hyperbolic condition  $ad - bc = 1$ . Both bounds are of the correct order of magnitude. The upper bound can be established by classical sieving techniques, while the lower one is conditional subject to the conjecture that interpolates between the Elliot–Halberstam conjecture and the Bombieri–Vinogradov theorem. We discuss how to obtain an unconditional analog of that result by considering square-free numbers instead of the primes as well as the numbers with a prescribed number of prime factors (that we try to make as small as possible). The involved techniques include semilinear sieving, the asymptotic sieve as well as recent results on square-free numbers in arithmetic progressions. This is work in progress.