

Hurwitz series ring, operations over sequences in rings and binomial type sequences

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Abstract

We present some properties of the Hurwitz series ring $H_R[[t]]$, for an integral domain R , with multiplicative identity and zero characteristic. We show a closed form for the invertible elements by means of the complete ordinary Bell polynomials, highlighting some connections with well-known transforms of sequences. Then we focus the attention on some special subgroups studying their properties and introducing a new transform of sequences that allows to see one of this subgroup as an ultrametric dynamic space. Finally, considering a commutative ring with identity R , we examine some operations which can be defined over the set H_R of sequences of elements in R . These operations can also give H_R the structure of a ring. Focusing on the binomial convolution product and the operation induced by the composition of exponential generating functions, we provide new relations between these operations and their invertible elements. Moreover, we introduce a novel isomorphism between H_R equipped with the componentwise sum and the set of the sequences starting with 1_R equipped with the binomial convolution product. Thanks to this isomorphism, we find an interesting method for characterizing and generating all the binomial type sequences.