# FINITELY GENERATED ABELIAN GROUPS OF UNITS

Ilaria Del Corso
Università di PISA

The study of the group of units of a ring is an old problem. The first general result is the classical Dirichlet's Unit Theorem (1846), which describes the group of units of the ring of integers $\mathcal{O}_K$ of a number field $K$, proving that $\mathcal{O}_K^* \cong C_{2n} \times \mathbb{Z}^g$ where $n \geq 1$ and $g$ is determined by the structure of the field $K$.

In 1940 G. Higman discovered a perfect analogue of Dirichlet's Unit Theorem for a group ring $\mathbb{Z}T$ where $T$ is a finite abelian group: $(\mathbb{Z}T)^* \cong \pm T \times \mathbb{Z}^g$ for a suitable explicit constant $g$.

In 1960 Fuchs in [Abelian Groups, (Pergamon, Oxford, 1960); Problem 72] posed the following problem.

> *Characterize the groups which are the groups of all units in a commutative and associative ring with identity.*

In the first part of my talk I will briefly presents the main results that have been found in this context along the years.

In the second part I will present my recent results. In two joint papers with R. Dvornicich, we restrict Fuchs' question to *finite abelian groups* we obtain necessary conditions for a group to be realizable as group of units of a ring. We also produce infinite families of both realizable and non-realizable groups.

In a very recent paper (JLMS 2019) I addressed Fuchs' question for *finitely generated abelian* groups characterizing those groups which arise in some fixed classes of rings, namely the integral domains, the torsion free rings and the reduced rings. This result is obtained via a very accurate study of the orders of cyclotomic $K$-algebras where $K$ is a suitable number field.

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