

An effective criterion for periodicity of ℓ -adic continued fractions

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It goes back to Lagrange that a real quadratic irrational has always a periodic continued fraction. Starting from decades ago, several authors ([Mah34], [Sch70], [Rub70] and [Bro01]) proposed different definitions of a p -adic continued fraction, and the definition depends on the chosen system of residues mod ℓ . It turns out that the theory of ℓ -adic continued fractions has many differences with respect to the real case; in particular, no analogue of Lagrange's theorem holds, and the problem of deciding whether the continued fraction is periodic or not seemed to be not known. In recent work with F. Veneziano and U. Zannier [CVZ18] we investigated the expansion of quadratic irrationals, for the ℓ -adic continued fractions introduced by Ruban [Rub70], giving an effective criterion to establish the possible periodicity of the expansion. This criterion, somewhat surprisingly, depends on the “real” value of the ℓ -adic continued fraction.

References

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