The distribution of imaginary parts of the nontrivial zeros of the Riemann zeta function

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Abstract

We consider the distribution of $(\alpha\gamma)$ modulo one, where α is a nonzero real number and γ runs over the imaginary parts of the nontrivial zeros of the Riemann zeta function. In 1975 Hlawka [3], improving the previous results by Rademacher and Elliott, proved that this sequence is uniformly distributed modulo one. Using Weyl's criterion, this means that, for suitable functions f, it holds

$$\sum_{0<\gamma\leq T} f(\alpha\gamma) = N(T)\int_0^1 f(x)\,dx + o(N(T)),$$

where $N(T)$ is the customary counting function for the nontrivial zeros. We will give some context to better understand this formula and we will present some improvements and generalizations obtained by Ford and Zaharescu [2] alone and then in collaboration with Soundararajan [1]. Finally, we will talk about possible ways in which these works could be further developed.

References

[1] K. Ford, K. Soundararajan, and A. Zaharescu, *On the distribution of imaginary parts of zeros of the Riemann zeta function. II*, Math. Ann., 343 (2009), pp. 487–505.

[2] K. Ford and A. Zaharescu, *On the distribution of imaginary parts of zeros of the Riemann zeta function*, J. Reine Angew. Math., 579 (2005), pp. 145–158.

[3] E. Hlawka, *Über die Gleichverteilung gewisser Folgen, welche mit den Nullstellen der Zetafunktion zusammenhängen*, Österreich. Akad. Wiss. Math.-Naturwiss. Kl. S.-B. II, 184 (1975), pp. 459–471.