Zeros of generalized Hurwitz zeta functions

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Abstract

In [3], Davenport and Heilbronn proved that the classical Hurwitz zeta function $\zeta(s, \alpha)$ has infinitely many zeros in the half-plane $\sigma > 1$, provided that $\alpha \notin \{1, \frac{1}{2}\}$ is rational or transcendental. The algebraic irrational case was later settled by Cassels [1]. In this talk, we consider a function $f(n)$ periodic of period $q \geq 1$ and a real number $0 < \alpha \leq 1$. We prove that the series $F(s, f, \alpha) = \sum_{n=0}^{\infty} \frac{f(n)}{(n+\alpha)^s}$ has infinitely many zeros for $\sigma > 1$, improving a recent result by Chatterjee and Gun [2]. We will also give a brief overview of the main results about zeros of Dirichlet series in the region of absolute convergence.

References

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[4] E. Saias, A. Weingartner, *Zeros of Dirichlet series with periodic coefficients*, Acta Arithmetica 140 (2009), 335-344.