

The greatest common divisor of n and the n -th term of a linear recurrence, and related problems

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The arithmetical relations between the terms u_n of a linearly recurrent sequence and their index n are a subject which has witnessed a number of new developments. We shall describe recent progress on two dual problems: (1) When does n divide the n -th Fibonacci number F_n ? (2) When is n coprime to F_n ? In both cases, the distribution of such n can be estimated quite precisely, and for question 2 one can even give an explicit formula for the asymptotic density of the n 's such that $\gcd(n, F_n)$ equals a fixed integer $k \geq 1$. Next, we shall see what happens for a generic linear recurrence u_n in place of F_n ; and then when n and u_n are replaced respectively by linear recurrences u_n and v_n , in which case the techniques belong to Diophantine analysis.

References

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