A rigidity theorem for translates of uniformly convergent Dirichlet series

Alberto Perelli - Università di Genova

Abstract: (joint work with Mattia Righetti) It is well known that the Riemann zeta function, as well as several other L-functions, is universal in the strip $1/2 < \sigma < 1$; this is certainly not true for $\sigma > 1$. Answering a question of Bombieri and Ghosh, we give a simple characterization of the analytic functions approximable by translates of L-functions in the half-plane of absolute convergence. Actually, this is a special case of a general rigidity theorem for translates of Dirichlet series in the half-plane of uniform convergence. Our results are closely related to Bohr's equivalence theorem.

References

[1] T.M.Apostol - *Modular Functions and Dirichlet Series in Number Theory* - Springer Verlag 1976.

[2] H.Bohr - *Zur Theorie der allgemeinen Dirichletschen Reihen* - Math. Ann. **79** (1918), 136–156.

[3] E.Bombieri, A.Ghosh - *Around the Davenport-Heilbronn function* - Russian Math. Surveys **66** (2011), 221–270.

[4] K.Chandrasekharan - *Introduction to Analytic Number Theory* - Springer Verlag 1968.

[5] H.Davenport, H.Heilbronn - *On the zeros of certain Dirichlet series* - J. London Math. Soc. **11** (1936), 181–185.

[6] J.Kaczorowski, A.Perelli - *Some remarks on the convergence of the Dirichlet series of L-functions and related questions* - Math. Zeitschrift **285** (2017), 1345–1355.

[7] A.A.Karatsuba, S.M.Voronin - *The Riemann Zeta-Function* - de Gruyter 1992.

[8] M.Kuniyeda - *Uniform convergence-abscissa of general Dirichlet's series* - Tôhoku Math. J. **9** (1916), 7–27.

[9] K.Matsumoto - *A survey on the theory of universality for zeta and L-functions* - In *Number Theory: Plowing and Starring Through High Wave Forms*, ed. by M.Kaneko *et al.*, p.95–144, World Scientific 2015.

[10] M.Righetti - *On Bohr's equivalence theorem* - J. Math. An. Appl. **445** (2017), 650–654; corrigendum ibid. **449** (2017), 939–940.

[11] M.Righetti - *On the density of zeros of linear combinations of Euler products for $\sigma > 1$* - to appear in Algebra and Number Theory.

[12] S.M.Voronin - *A theorem on the "universality" of the Riemann zeta-function* (Russian) - Izv. Akad. Nauk SSSR Ser. Mat. **39** (1975), 475–486. English transl. Math. USSR-Izv. **9** (1975), 443–453.