Arithmetic random waves and lattice points on spheres.

Riccardo Walter Maffucci
King's College London/University of Oxford

We consider random Gaussian eigenfunctions of the Laplacian on the 3-dimensional torus ('arithmetic random waves'), and discuss the number of nodal intersections against a straight line segment. The case of nowhere zero curvature curves was investigated by Rudnick, Wigman and Yesha.

The expected intersection number, for a generic smooth curve, is universally proportional to the length of the curve, times the wavenumber, independent of the geometry. The main focus of the talk will be the nodal intersections variance in the straight line case. We give upper bounds for the variance (in the high-energy limit), depending on the arithmetic properties of the straight line.

The problem is closely related to the theory of lattice points on spheres. We will discuss bounds for the number of lattice points lying in specific regions of a sphere, relying on Diophantine approximation.

[1] Bourgain, Jean, and Rudnick, Zeév, "Restriction of toral eigenfunctions to hypersurfaces and nodal sets", Geometric and Functional Analysis 22.4 (2012): 878-937.

[2] Maffucci, Riccardo W. "Nodal intersections for random waves against a segment on the 3-dimensional torus." Journal of Functional Analysis 272.12 (2017): 5218-5254.

[3] Rudnick, Zeév, Wigman, Igor and Yesha, Nadav, "Nodal intersections for random waves on the 3-dimensional torus", Ann. Inst. Fourier (Grenoble) 66 (2016), no. 6, 2455–2484.