ON THE GREATEST COMMON DIVISOR OF n AND THE nTH FIBONACCI NUMBER

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Let $\mathcal{A}$ be the set of all integers of the form $\gcd(n, F_n)$, where n is a positive integer and F_n denotes the nth Fibonacci number. We prove that

$$\#\left(\mathcal{A} \cap [1, x]\right) \gg x / \log x$$

for all $x \geq 2$, and that

$$\#\left(\mathcal{A} \cap [1, x]\right) = o(x)$$

as $x \to \infty$, see [3]. This is a joint work with Carlo Sanna.

As a consequence, we obtain that the set of all integers n such that n divides F_n has zero asymptotic density relative to $\mathcal{A}$. Related results were given in [1, 4, 5].

The proofs rely on a recent result of Cubre and Rouse [2] which gives, for each positive integer n, an explicit formula for the density of primes p such that n divides the rank of appearance of p, that is, the smallest positive integer k such that p divides F_k.

References

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