Low discriminants for number fields of degree 8 and signature (2,3)

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Let K be a number field of degree n, with discriminant d_K, and let r_1 be the number of real embeddings of K in $\mathbb{C}$ and r_2 be the number of couples of complex embeddings, so that $n = r_1 + 2r_2$.
A classical problem asks to establish the minimum value for $|d_K|$ when K ranges in the set of fields with a fixed signature (r_1, r_2). During the last century many methods for answering the question were set: from the classical tools of Geometry of Numbers invented by Minkowski to the analytic estimates involving the Dedekind Zeta functions, due to Odlyzko [2], Poitou [5] and Serre [6] up to the algorithmic procedures, based on number-geometric ideas, developed by Pohst [3], Martinet [1] and Diaz y Diaz [4] (in collaboration with the previous authors): with these ides the problem was solved for $n \leq 7$, with any signature, and also for $n = 8$, if the signature is either $(8, 0)$ or $(0, 4)$.
In this work we exploit the methods aforementioned in order to prove the following results:

Theorem 1. *Let d_K be the discriminant of a number field K with degree 8 and signature $(2, 3)$. Then the minimum value of $|d_K|$ is equal to 4286875.*

Theorem 2. *There are 56 number fields of degree 8 and signature $(2, 3)$ with $|d_K| \leq 5726300$; with the exception of two non-isomorphic fields with $|d_K| = 5365963$, every field in the list is uniquely characterized by the value of $|d_K|$.*

References

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